Name Solutions

Section: (check one) Jiao ____ Melloch____

October 30, 2008

ECE 311 Exam 2 Fall 2008

Closed Text and Notes

1) Be sure you have 14 pages.

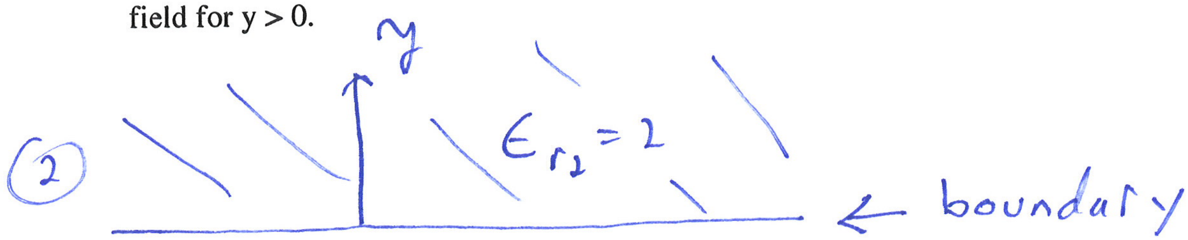
2) Write only on the question sheets. Show all your work. If you need more room for a particular problem, use the reverse side of the same page.

4) Write neatly, if your writing is illegible then print.

5) The last 4 pages contain equations that may be of use to you.

6) This exam is worth 100 points.

(12 pts) 1. The boundary between two dielectrics is the xz-plane. For y > 0 $\varepsilon_{r2} = 2$. For y < 0, $\mathbf{E}_1 = (2\mathbf{a}_x - 3\mathbf{a}_y + 6\mathbf{a}_z)x10^4 \frac{V}{m}$ and $\mathbf{P}_1 = (1.77\mathbf{a}_x - 2.66\mathbf{a}_y + 5.31\mathbf{a}_z)x10^{-7} \frac{C}{m^2}$. Find $\mathbf{E}_2$ the electric field for y > 0.



(1) $\vec{E}_1 = (2\hat{a}_x - 3\hat{a}_y + 6\hat{a}_z) \times 10^4 \frac{V}{m}$

$\vec{P}_1 = (1.77\hat{a}_x - 2.66\hat{a}_y + 5.31\hat{a}_z) \times 10^{-7} \frac{C}{m^2}$

$$\vec{E}_{1T} = (2\hat{a}_x + 6\hat{a}_z) \times 10^4 \frac{V}{m} = \vec{E}_{2T}$$

$$\vec{E}_{1N} = -3\hat{a}_y \times 10^4 \frac{V}{m} \qquad \vec{P}_{1N} = -2.66 \times 10^{-7} \frac{C}{m^2}$$

$$\vec{D}_{1N} = \epsilon_0 \vec{E}_{1N} + \vec{P}_{1N} = \left[(8.854 \times 10^{-12} \tfrac{F}{m})(-3 \times 10^4 \tfrac{V}{m}) - 2.66 \times 10^{-7} \tfrac{C}{m^2}\right]\hat{a}_y$$

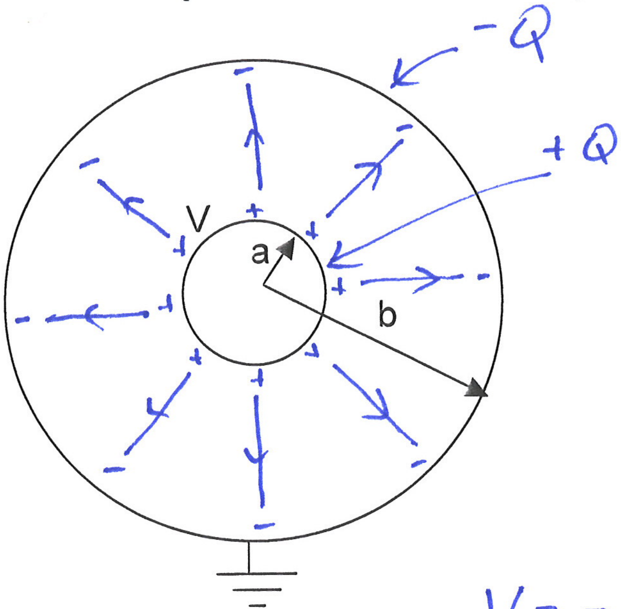
$$= -5.32 \times 10^{-7} \hat{a}_y \frac{C}{m^2} = \vec{D}_{2N}$$

$$\vec{E}_{2N} = \frac{\vec{D}_{2N}}{\epsilon_{r_2}\epsilon_0} = \frac{-5.32 \times 10^{-7} \frac{C}{m^2}}{2(8.854 \times 10^{-12} \frac{F}{m})}\hat{a}_y = -3 \times 10^4 \frac{V}{m}\hat{a}_y$$

$$\vec{E}_2 = \vec{E}_{2T} + \vec{E}_{2N}$$

$$\vec{E}_2 = (2\hat{a}_x - 3\hat{a}_y + 6\hat{a}_z) \times 10^4 \frac{V}{m}$$

(12 pts) 2. Determine the capacitance of two co-centric spheres as shown of radius a and b. Let the outer sphere be at 0 V and the inner sphere at potential V.



$$\vec{D} = \frac{Q}{4\pi r^2}\hat{a}_r, \quad a < r < b$$

$$\vec{E} = \frac{\vec{D}}{\epsilon_0} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{a}_r, \quad a < r < b$$

$$V = -\int_b^a \frac{Q}{4\pi\epsilon_0 r^2}\hat{a}_r \cdot dr\,\hat{a}_r$$

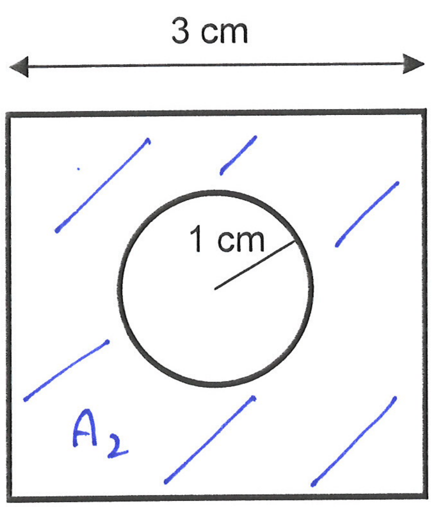
$$V = -\int_b^a \frac{Q}{4\pi\epsilon_0 r^2}dr = \left.\frac{Q}{4\pi\epsilon_0 r}\right|_b^a$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)}$$

$$C = \frac{4\pi\epsilon_0}{\left(\frac{1}{a} - \frac{1}{b}\right)} = \frac{4\pi\epsilon_0}{\left(\frac{b-a}{ab}\right)} = \frac{4\pi\epsilon_0 ab}{b-a}$$

(12 pts) 3. An object is 10 cm long with a square cross-section of 3 cm x 3 cm. The object is made of two materials. A cylinder of 1 cm radius, 10 cm long of conductivity 1 S/m surrounded by a material of conductivity 0.5 S/m. The cross-section is shown in the figure. If 10 volts is applied across the length of the object, what current is flowing?



$$E_1 = E_2 = \frac{V}{l} = \frac{10V}{0.1m} = 100\frac{V}{m}$$

$$J_1 = \sigma_1 E_1 = \left(1\frac{S}{m}\right)\left(100\frac{V}{m}\right) = 100\frac{A}{m^2}$$

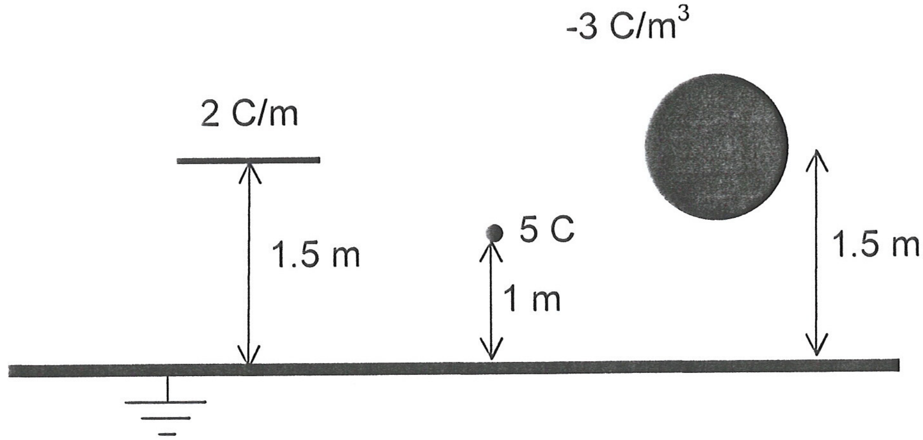
$$J_2 = \sigma_2 E_2 = \left(0.5\frac{S}{m}\right)\left(100\frac{V}{m}\right) = 50\frac{A}{m^2}$$

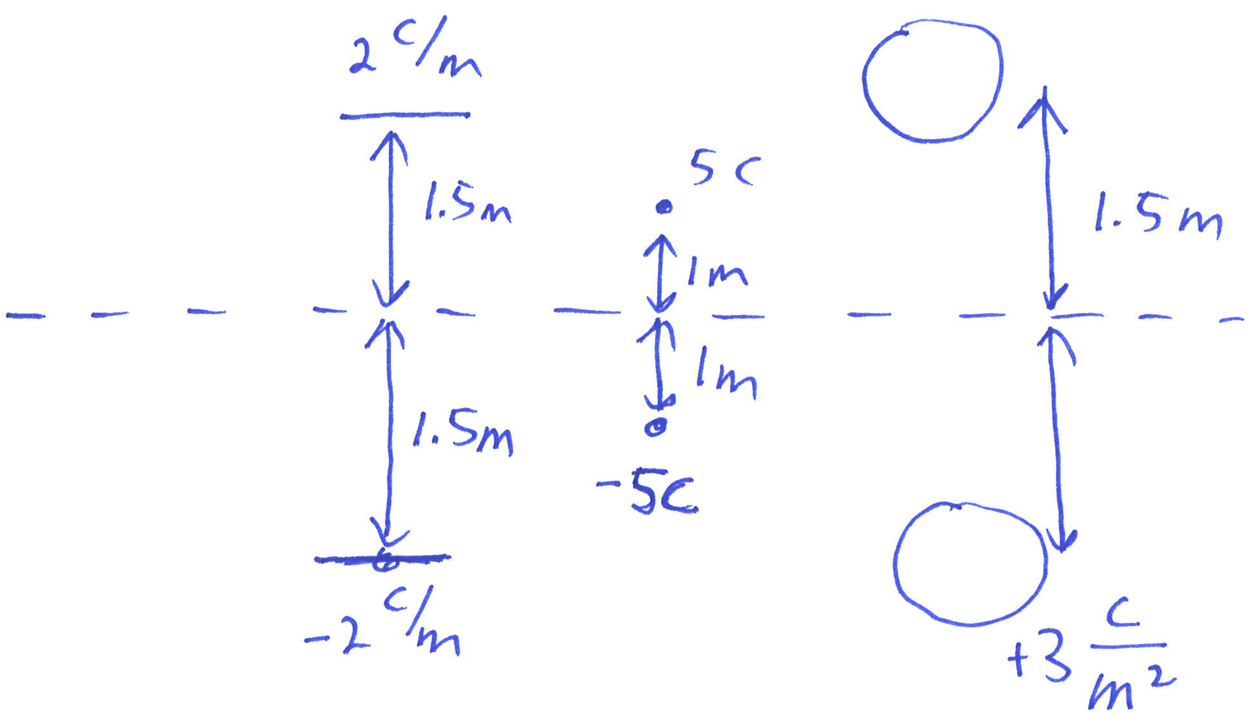
$$I_1 = J_1 \pi r^2 = \left(100\frac{A}{m^2}\right)(\pi)(0.01m)^2 = 0.0314\,A$$

$$I_2 = J_2 A_2 = \left(50\frac{A}{m^2}\right)\left[(0.03m)^2 - \pi(0.01m)^2\right] = 0.0292\,A$$

$$I = I_1 + I_2 = 0.0314A + 0.0292A = 0.0607A$$

(12 pts) 4. draw the system of image charges that could be used to find the voltage and electric field above the grounded infinite plane.

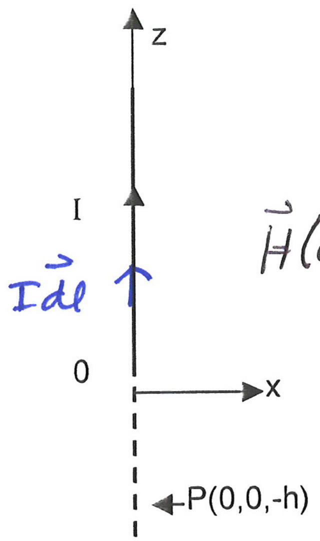




(6 pts) 5. For $-1<x<1$ and $-1<y<1$ a current density of $\mathbf{J} = (2\mathbf{a}_z)x10^{-6}\frac{A}{m^2}$ is flowing. Over the surface of a sphere of radius 1m centered at the origin, what is $\oint \mathbf{B} \bullet \mathbf{ds}$?

$$\oint \vec{B} \cdot \vec{ds} = 0$$

(10 pts) 6. A semi-infinite line current carries a current of I A along the z-axis from the origin at z = 0 to z = infinity. Assuming free space everywhere else calculate the magnetic filed intensity at any point P(0,0,-h) on the negative z-axis.



$$I\vec{dl} = I\,dz\,\hat{a}_z \qquad \hat{a}_R = -\hat{a}_z$$

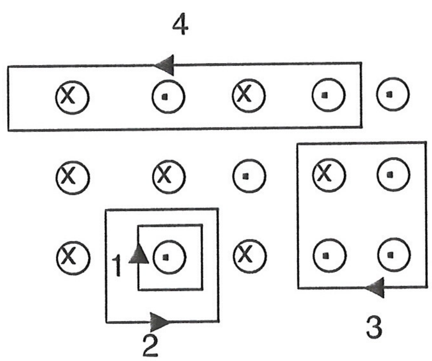
$$\vec{R} = -(z+h)\,\hat{a}_z$$

$$\vec{H}(0,0,-h) = \int \frac{I\vec{dl} \times \hat{a}_R}{4\pi R^2} = \int_0^{\infty} \frac{I\,dz\,\hat{a}_z \times (-a_z)}{4\pi\,(z+h)^2}$$

so,

$$\vec{H}(0,0,-h) = 0 \qquad \text{since } \hat{a}_z \times \hat{a}_z = 0$$

(12 pts) 7. Determine $\oint \mathbf{H} \bullet \mathbf{dl}$ over the paths shown. Note a dot signifies a filament carrying1A of current out of the page and an X a filament carrying 1 A of current into the page.



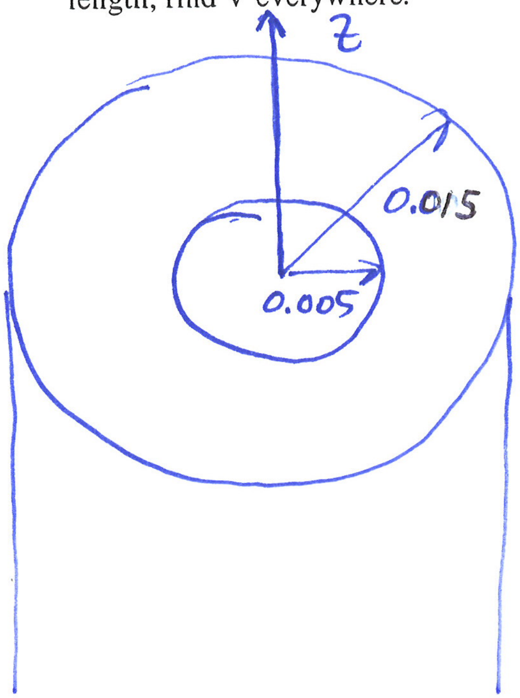
$\oint_1 \mathbf{H} \bullet \mathbf{dl} =$ $-1\ A$

$\oint_2 \mathbf{H} \bullet \mathbf{dl} =$ $+1\ A$

$\oint_3 \mathbf{H} \bullet \mathbf{dl} =$ $-2\ A$

$\oint_4 \mathbf{H} \bullet \mathbf{dl} =$ 0

(12 pts) 8. A cylindrical capacitor has inner radius of 0.005 m and outer radius 0.015 m. The outer cylinder is grounded and 10 V is applied to the inner cylinder. Assuming the capacitor is infinite in length, find V everywhere.



$V(0.005m) = 10V$

$V(0.015m) = 0V$

$\nabla^2 V = 0$

$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$

due to symmetry, V won't vary with z or ϕ

so, $\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) = 0 \Rightarrow \frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) = 0$

$\rho \frac{\partial V}{\partial \rho} = a \Rightarrow \frac{\partial V}{\partial \rho} = \frac{a}{\rho} \Rightarrow \partial V = a\frac{\partial \rho}{\rho}$

$V(\rho) = a \ln(\rho) + b$

$V(0.015m) = 0 = a \ln(0.015) + b$

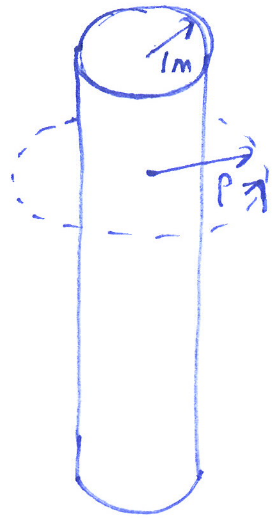
$b = -a \ln(0.015)$

$V(\rho) = a \ln \rho - a \ln(0.015) = a \ln \frac{\rho}{0.015}$

$V(0.005) = 10V = a \ln \frac{0.005}{0.015} \Rightarrow a = \frac{10V}{\ln\left(\frac{1}{3}\right)}$

$V(\rho) = \frac{10V}{\ln\left(\frac{1}{3}\right)} \ln \frac{\rho}{0.015} = -9.1 \ln \frac{\rho}{0.015} = -9.1 \ln \rho - 38.22$

(12 pts) 9. An infinitely long, hollow cylindrical shell of radius 1 m has $\mathbf{K} = (2\mathbf{a}_z)x10^{-6}\frac{A}{m}$ flowing on its outer wall. Determine H everywhere.



From symmetry H will not depend on z or ϕ and will only have a component in the $\hat{a}_\phi$ direction

for $\rho < 1\,m$ $\oint \vec{H}\cdot d\vec{l} = I_{enclosed} = 0$

$$\oint \vec{H}\cdot d\vec{l} = H_\phi 2\pi\rho = 0$$

so $\vec{H} = 0$ for $\rho < 1\,m$

for $\rho > 1\,m$ $\oint \vec{H}\cdot d\vec{l} = I_{encl.} = K_z 2\pi(1m)$

$$H_\phi 2\pi\rho = \left(2\times10^{-6}\frac{A}{m}\right)2\pi(1m)$$

$$H_\phi \rho = 2\times10^{-6} A$$

$$H_\phi = \frac{2\times10^{-6} A}{\rho}$$

$$\vec{H} = 0, \quad \rho < 1m$$

$$\frac{2\times10^{-6} A}{\rho}\hat{a}_\phi, \quad \rho > 1m$$